Causality, Time flow and Abstraction in Networks

Kausalität, Zeit und Abstraktion in Netzwerken

A Proposal data

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B Project Description

1 State of the art and preliminary work

Networks

In the first half of the previous century, McCulloch and Pitts introduced the original model of Boolean Automata Network (BAN) [1]. It famously paved the way for an exceptionally prolific heritage including Finite Automata Theory [2–6], (Artificial) Neural Networks [7–10], branches of Cognitive Neuroscience [11, 12], and other Biological Regulation Systems Modelling [13, 14]. Today, at the intersection of (F1) the "applied" field that conceives mathematical models of biological regulation networks, and (F2) the "theoretical" field that studies and develops computational models, lies an opportunity to take a reunifying perspective on networks. Biological networks modelling (F1) alike Automata Theory (ꟻ2) takes interest in networks. F1 focuses on what networks do in the sense of how they behave. ꟻ2 focuses more on what networks do in the sense of what they produce. F1 studies systems comprised of networking units called "automata" and puts definite emphasis on their network-ness, on the events that take place inside the systems, affecting the automata, and on the relations between these events. ꟻ2 rather puts emphasis on the attendant notions of computation and computational capacity – more precisely on the *outcome* of computation (formal languages and their recognition) and on computational capacity measured *in terms of outcomes*. ꟻ2 saves the word "automaton" to denote the computing unit (which is a network) as a whole. The research I propose assumes *a kind of computational capacity that is attributable precisely to the network-ness*. It aims at formalising a notion of computational capacity informing on behavioural possibilities of networks that are not designed to produce one specific outcome. For this, a raw version of the BAN formalism provides an especially favourable and general framework.

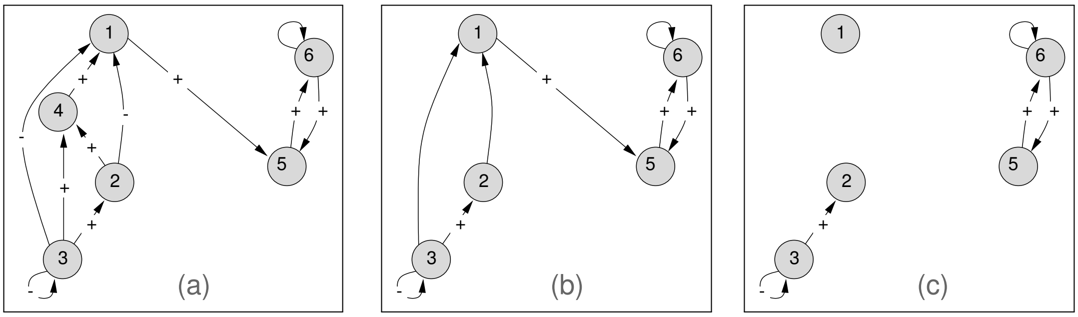
My research draws its propitousness from a specific approach to BANs. Because of that, the presentation of my project needs to introduce and discuss a range of concepts. This risks making the presentation overly non-figurative. To avoid this, I will compensate with some examples. The one below simultaneously (i) clarifies the project's structuring concerns and (bibliographical) setting, and (ii) introduces the main concepts of the BAN formalism.

Motivating Example

Let $\mathcal{N}$ be the name of the BAN represented in Fig.1 (a) : $\mathcal{N}$ is comprised of $n = 6$ automata, namely $1, 2, 3, 4, 5$ and 6 (or ①, ②, ③, ④, ⑤, and ⑥) whose behaviours are defined by the Boolean local update functions of type $\mathbb{B}^n = \{0,1\}^n \to \mathbb{B} = \{0,1\}$ listed below Fig.1 (a), respectively : f_1, f_2, f_3, f_4, f_5, and f_6. This means that starting in an arbitrary configuration $x = (x_1, x_2, \ldots, x_6) \in \mathbb{B}^n$ of $\mathcal{N}$, if, for instance, ② is updated, then this automaton takes (or remains in) state $f_2(x)$. It takes state $f_2(x) = \neg x_2$ if it is *unstable* in x, *i.e.* if $2 \in U(x) = \{i \leq n : f_i(x) \neq x_i\}$. It remains in state $x_2 = f_2(x)$ if it is *stable* in x, *i.e.* if $2 \in S(x) = \{i \leq n : f_i(x) = x_i\}$. And if no other automaton is updated in x, then the whole BAN $\mathcal{N}$ moves from configuration x to configuration x' where $\forall i \neq 2,\ x'_i = x_i$ and $x'_2 = f_2(x)$. If however, ⑥ is also updated in x, then $\mathcal{N}$ moves to configuration x' where $\forall i \notin \{2, 6\},\ x'_i = x_i$ and $x'_2 = f_2(x),\ x'_6 = f_6(x)$.
Importantly – here – $\mathcal{N}$ is *not* a dynamical system : $\mathcal{N} = \{f_i : \mathbb{B}^n \to \mathbb{B} \,|\, i \leq n\}$. It is a set of local networked mechanisms that say nothing about how these mechanisms are, nor may be set in time. This distinguishs : (i) a notion of causality that relates the clockworks of the network strictly to the possibilities of changes that they are responsible for, from (ii) an independent notion of time flow responsible for the relative arrangement of occurrences of possible events [15] [1].
In the literature [9, 16–34] [35–42], Automata Networks (ANs) are usually defined as, or taken to represent dynamical systems. They are used to study system dynamics or sets of possible system

1. Green citations in the text refer to publications of my own.

dynamics. And this choice fuses the underlying notion of causality with that of time. As a consequence, the AN definition is usually made to imply specific updating constraints unlike the one given in above. Moreover, the discrete dynamical systems view on BANs tends to imply a distinction between (1) process of change and (2) (result of) change – where implicitly, a (result of) change is either what we observe as a consequence of a process of change, or it is an approximation or abstraction of a process of change. On the contrary, here, for the sake of studying *time, causality and abstraction ensuing from observation*, a less abstract view on the formalism is taken. The distinction is not made : change is anything that has observable consequence, if only the observable consequence of our noticing it. Process of change is change if it has observable consequence in itself. If it hasn't, then it isn't something there is anything to be said about. The important consequence of this is to shift the focal point from the *states* of automata, to the *changes* of automata states. So under a dynamical systems view, BANs are just another interaction system model. But with this project's view on change, the BAN formalism becomes a convenient framework in which we can deliberately study the laws and principles that an arbitrary object (formal or natural) obeys *because* the concept of interaction system applies to it.



$$\begin{cases} f_1 : x \mapsto x_4 \wedge (\neg x_2 \vee \neg x_3) \\ f_2 : x \mapsto x_3 \\ f_3 : x \mapsto \neg x_3 \\ f_4 : x \mapsto x_2 \vee x_3 \\ f_5 : x \mapsto x_1 \vee x_6 \\ f_6 : x \mapsto x_6 \vee x_5 \end{cases} \qquad \begin{cases} f_1 : x \mapsto x_2 \oplus x_3 \\ f_2 : x \mapsto x_3 \\ f_3 : x \mapsto \neg x_3 \\ \\ f_5 : x \mapsto x_1 \vee x_6 \\ f_6 : x \mapsto x_6 \vee x_5 \end{cases} \qquad \begin{cases} f_1 : x \mapsto 0 \\ f_2 : x \mapsto x_3 \\ f_3 : x \mapsto \neg x_3 \\ \\ f_5 : x \mapsto x_6 \\ f_6 : x \mapsto x_6 \vee x_5 \end{cases}$$

Fig. 1 – TOP : Digraphs $G = (V, A)$ representing BAN interaction structures ($V = \{i \leq n\}$). BOTTOM : Sets of Boolean functions representing BAN clockworks. (a) A monotone BAN $\mathcal{N}$. (b) The non-monotone version of $\mathcal{N}$ that we see if we ignore automaton 4 and if automaton 4 happens to be systematically updated before automaton 1 is. (c) The version of $\mathcal{N}$ that we would see if on top of that, automata 2 and 3 were caught in the same rhythm as described in (3) .

Imagine that there actually exists a real system in nature that works exactly as the BAN $\mathcal{N}$ of Fig.1 (a) does. We call it $\mathcal{N}$ too. And imagine that we human observers of reality are actually observing this real system $\mathcal{N}$ in action. And at the time we are doing that, for some reason, parts of $\mathcal{N}$ are behaving rhythmically. In particular, state changes of ① and ④ are happening at the same frequency, although with some phase offset, so that everything is happening exactly as if ④ was systematically updated immediately before ① is.

Imagine that because of the specific level of abstraction or detail with which we observers are considering $\mathcal{N}$, we are unaware of ④'s existence – N.B. This does not necessarily imply a default in our observation, it could be the consequence of something in relation to ④ not being appreciable at the particular level of abstraction we have chosen to look at $\mathcal{N}$, given the specific attributes of $\mathcal{N}$ that we are interested in and focusing on (consider having the *change* of ④ represent the rapid decoding of mRNA, and changes of other automata represent slower processes such as protein syntheses). As a consequence, every time we witness ① change states, ④ just has. And, while $\mathcal{N}$ follows trajectory : $\ldots \longrightarrow x = (x_1, \ldots, x_4, \ldots) \xrightarrow{4} x' = (x_1, \ldots, f_4(x), \ldots) \xrightarrow{1} x'' = (f_1(x'), \ldots, f_4(x), \ldots) \longrightarrow \ldots$ where $f_1(x') = f_4(x) \wedge (\neg x_2 \vee \neg x_3) = (x_2 \vee x_3) \wedge (\neg x_2 \vee \neg x_3) = x_2 \oplus x_3$, we observers just see : $\underline{x} = (x_1, \ldots\ldots) \xrightarrow{1} \underline{x}'' = (f_1(x'), \ldots\ldots)$ where $\underline{x} \in \mathbb{B}^{n-1}$ denotes the restriction of $x \in \mathbb{B}^n$ to $\{i \leq n,\, i \neq 4\}$. And what is given of $\mathcal{N}$ for us to understand is illustrated in Fig.1 (b) where

① depends non-monotonously on ② (and ③). This means that both x_2 and $\neg x_2$ appear in the conjunctive normal form of $f_1(x)$ so arc $(2,1)$ in the interaction digraph G (*cf.* top of Fig.1) cannot be given a sign. It would be given a $+$ sign (resp. a $-$ sign) if $f_1(x)$ only depended on x_2 (resp. $\neg x_2$).
In the literature, wherever BANs are considered as stand-alone mathematical objects, it is customary to restrict the f_i functions to a certain class of functions [17, 21, 23, 43–47] [35, 36, 48, 49], and thoroughly study the BANs' behaviours under these conditions. A typical example is the restriction to functions that are expressible in terms of a limited number of logical connectors such as the $\oplus$ (XOR) connector which makes the BAN non-monotone [50, 51] [48, 52]. On the contrary, when BANs are considered as potential models of biological (genetic) regulation networks, BANs tend to be assumed to be monotone. [26, 29, 53–55]. The previous paragraph illustrates how it suffices to have different time scales ruling the changes in a system for us observers to be seeing a non-monotone BAN where under different conditions of observation and timing, we could be seeing a monotone BAN.
Building further on the same example, imagine that the behaviours of ② and ③ are also caught in the same rhythm as ① (and ④) and everything is happening as if automata updates were being made in the following order : $\dots\ 3,2,4,1,3,2,4,1,3,2,4,1\ \dots$ With this rhythmical arrangement of changes, ① is stuck in state $x_1 = 0$. So under these conditions of observation and timing, we have no reason to suspect that ① receives any influences from the rest of the network, nor that it has any influence on it. What is given of $\mathcal{N}$ for us to understand is illustrated in Fig.1 (c). And yet if one automaton were to change pace, slow down for instance, even if only momentarily, then ① might at some point take state 1. The oscillations of ③ might even spread to ⑤. Or, if ⑤ and ⑥ had been locked in state 0 until then, ① might unlock ⑤ which in turn might generate the irrevocable effect of allowing ⑥ to take state 1. None of this could we foresee, nor even understand if it occurred, because of the understanding we have already built of $\mathcal{N}$ under the given conditions of observation and timing of $\mathcal{N}$.
In the literature, BANs are sometimes taken as possible models of real networks supposed to be built in consistency with biological data and *in consideration of the risk that this data might be incomplete* [56–59]. In these contexts, considering a real network $\mathcal{R}$, considering that digraph $G = (V, A)$ models its structure of interactions as in the top of Fig. 1, considering also that two arbitrary entities/automata/genes/... E_i and E_j of $\mathcal{R}$ are represented by ⓘ and ⓙ in the formal model $\mathcal{M}$ of $\mathcal{R}$, three cases are considered : (c1) Ei "really" has an (indirect) influence on Ej and $\mathcal{M}$ takes this into account, *i.e.* $(i,j) \in A$, (c2) Ei "really" has no influence on Ej and $\mathcal{M}$ takes this into account, *i.e.* $(i,j) \notin A$, or else (c3) Ei "really" has an influence on Ej but the data having failed to evidence this fact, $\mathcal{M}$ is failing to represent it also, *i.e.* arc (i,j) is unintentionally missing from G. This is where bioinformaticians can help biologists deal with "their faulty work material" when it is also too complex to be easily tractable. With formal methods, they build tools to check the consistency of biological models with biologists' observations and interpretations of the underlying data, and sometimes also to provide collections of models consistent with it [56, 59–64].
The running BAN example $\mathcal{N}$ with the assumptions of the previous paragraph shows that : (i) Case (c2) might not make any sense at all beyond a very specific level of abstraction that fixes temporal and observational conditions, and (ii) there are other reasons for an arc (i,j) to be missing from G than "the unfortunate limits of biologists' clearance to biological reality" underpinning Case (c3).

Abstraction

The example as a whole shows how anodyne temporal and observational conditions can subtly but decisively impact on the understanding of a system that we observers build for ourselves, and even before that, on our very defining of the system, what is in it, what is not. One might argue that *this always happens : there always is a risk of missing out information by not looking closely or often enough*. But lack of information is not a bad thing in itself. All lacks of information are not equivalent. Some can be exploited rather than avoided. The properties we attribute to a system depend on what we assume is part of the system and what we assume isn't. And in turn, how we define the system depends on the level of abstraction at which we consider it before we formalise a representation of it. Abstraction in that sense affects the information we use. It is a useful, necessary, self-imposed restriction on our access to causality. The abstraction inherent to the formal description of an object or property coincides with a special kind of lack of information about that object/property. This special kind of lack of information stands for possibilities of *finer* (rather than *missing*) descriptions of the object/property. If identified, it can be used to direct further exploration of the object/property (*cf.* below). My project offers to formally domesticate the neighbouring notions of *abstraction* and *representation* (see in particular A7 and C7 on pages 14 and 16). Additionally, BANs offer a simple enough formalism to consider lack of absolute certainty about the object descriptions we manipulate, and to study the range of underlying possibilities. So for the most part (*cf.* Plan A), my research examines properties of BANs under the assumption of exploitable lack of information.

Causality

It suffices to consider that automata "influence" one another to introduce a notion of causality. And it is very natural to interpret an arc $(i, j) \in A$ of the BAN's interaction structure G as meaning "i can cause j to change states". Yet there is no universal nor even consensual scientific notion of causality. This project doesn't aim at providing one. The notion is much too large and diverse to be pinned down to one fixed formal meaning. Besides, I believe causality stands for humans' instinctive way of grasping the world, and is an essential part of our motivation to explore the world further and have more of it grasped. It makes sense that the notion of causality serve science informally. Rather than sidelining intuitive instincts we can advantageously supervise their interference with scientific formalism. My research consists in doing that (*cf.* sequel).

Of course, BANs can be studied with purely mathematical interests and perspective. Then, causality is not such an important concern. Implication is enough. But because it applies only to specific properties of specific systems in specific conditions, implication has the downside of being much less portable than causality. And in theory, we are free to pick any restriction on the kinds of BANs we consider, as long as it helps us make progress towards new theoretical results. In some (asynchronous) contexts [27,65], the global update function $F : x \mapsto (f_1(x), \dots, f_n(x))$ (*cf.* (3)) is used to define a BAN. This definition is equivalent to the one chosen above. But it makes it more natural to pick restrictions on BANs that are given by properties of F, *e.g.* its non-expansivity : $\forall x, y \in \mathbb{B}^n : |\{x_i \neq y_i\}| \geq |\{F(x)_i \neq F(y)_i\}|$. Intuitively, F's non-expansivity corresponds to the BAN having a form of global instantaneous potential. Assuming F's non-expansivity happens to favour the derivation of some results about asynchronous BANs [66] [37] whose dynamical constraints precisely forbid them the use of this global potential. However mathematically sound are the mathematical results we prove thanks to mathematical assumptions/restrictions, disregard for intuitive causality stakes the applicability of those results. Without deliberate care, there is no reason to believe that we owe the deriving of these results to some deeper opportune relevance of the mathematical assumptions/restrictions. There is no reason to believe there is anything in those assumptions/restrictions that could enable the generalisation of the results beyond the setting they define, nor anything that could at least guarantee the relatability of the results to other existing results. The primary reason why we might have managed to derive anything under a particular restriction might be that it is an extremely strong restriction. It might be like studying crows by concentrating on the class of crows that a human being has reported seeing picking up a piece of pink plastic wrapper. It might be quite unclear what it is that we are studying and learning about exactly : the original (mathematical) object of interest ? the restriction ? Then, the only hope to actually build a global understanding of networks lies in the platonic wager that it will necessarily "emerge" from the accumulation of independent studies made of particular models of networks, juxtaposed for comparison. By focusing primarily on general – *i.e.* fundamental and thereby trans-disciplinary – network attributes (*e.g.* synchronism, non-monotony, reversibility, subsequence) rather than on network-specific properties (*e.g.* particular interaction digraphs, particular f_i's) or restrictions, and by studying rigorously these attributes' involvement in network behavioural possibilities, what we know of networks and what we don't can be re-examined and clarified. My research project offers to do that (*cf.* Section 2.5).

The next section discusses a typical example of fundamental, transdisciplinary notion that requires no specialised knowledge to be dealt with and made sense out of, but to be interpreted correctly, calls for precise in-depth understanding of both the inner workings and the expressivity of the formalism we apply it to. Developing this sort of understanding to enable the sound manipulation of fundamental, transdisciplinary notions is an essentially transdisciplinary task that Computer Scientists are in the ideal position to initiate, and that my project contributes to.

Formal synchronism and informal simultaneity

In each configuration $x \in \mathbb{B}^n$ of a BAN $\mathcal{N}$, the set $U(x)$ (*cf.* page 1) determines the set of local changes that are synchronously possible in x : $i \in U(x)$ means that $x_i \rightsquigarrow \neg x_i$ is a local change that is possible in x. The parallel update schedule (π) of a (B)AN $\mathcal{N}$ is the deterministic update schedule that maximally exploits synchronous possibilities of change. π imposes that *all* automata of $\mathcal{N}$ be updated systematically in each of its configurations, and thus that all automata systematically change states if they can. If $x = x(t) \in \mathbb{B}^n$ represents $\mathcal{N}$'s current configuration, then $x(t+1) = F(x) = (f_1(x), \ldots, f_n(x)) \in \mathbb{B}^n$ represents its next configuration. When this makes some automata react more quickly than we would like them to, intermediary automata can simply be added. This was originally done in the McCulloch and Pitts BANs. Since then, applications of (B)ANs have been going to and fro models of neural networks, [1, 9] models of statistical physical systems [8, 67] and models of genetic regulation [13, 14]. For some reason possibly related to this constant historical reprocessing of BANs, a surprisingly great many occidental modellers of biological regulation networks now confuse π with the notion of synchronism [22,62,64,68–79]. Asynchronism, to which π is wrongly opposed, is the update constraint that rules out the possibility of having *more than one* automaton be updated in a configuration. Synchronism (non-asynchronism) is the possibility of having more than one automaton be updated in a configuration. Its confusion with π has two consequences : (1) the neglect of all intermediary updating possibilities which neither rule out asynchronism altogether nor rule out synchronism altogether (in the case of BAN $\mathcal{N}^*$ of Fig. 2, this means disregarding $\mathcal{O}(2^{n+2^n})$ alternative transitions graphs), and (2) a framework in which synchronism can never be considered independently of all other very strong characteristic features of π (*e.g.* determinism, periodicity).

In the field of Genetic Regulation Networks Modelling With ANs, two additional assumptions about synchronism commonly motivate an asynchronous updating of automata states, and aggravate the disregard formal synchronism owes to its misguided association with the notion of determinism : (i) simultaneity in nature is highly fortuitous, (ii) simultaneity in nature is implicitly mapped one-to-one to synchronism in ANs [80]. Under such circumstances, invested interest in synchronism *per se* is extremely unlikely. And some problems – among those I have addressed [81] or propose to address – have little if any chances of arising.

To match an informal assumption about reality to a formal assumption in a model is not necessarily the same thing as *to formalise the informal assumption*. This is because of the other things the model already is a formalisation of. Until assumptions are merely matched to one another, and the interpretation of the model remains dissociated from the formalisation that initially defined it and set its expressivity, the sort of causality conveyed by the model can only at best be deliberate in the sense of expected. This stakes the soundness of the information drawn out of the modelling. For instance, when informal assumptions lead us to disregard altogether the possibility of the formal synchronism allowed for by our own (B)AN models, they make us disregard the following crucial fact and tacitly replace it by its exact opposite.

The possibility that several events can happen synchronously conveys an absence of causal relation between these events. Actually, the synchronism referred to here is one that conveys the absence of *any* kind of relation other than essentially temporal (unlike Concurrency's synchronism [82, 83]) Synchronously possible events are exactly events that don't need one another to occur. In any of BAN $\mathcal{N}^*$'s configurations (*cf.* Fig. 2) : (a) all 153 of its automata are unstable ($\forall x \in \mathbb{B}^{153}$: $U(x) = V^*$) and (b) there are 153 different local events ($x_i \rightsquigarrow f_i(x) = \neg x_i$) that are synchronously possible. All 153 instabilities (resp. events) are independent from one another. To impose an asynchronous updating of automata states is to assume that each event can prevent each other event from happening (or else that the model conveys the causality that we expect very poorly). In asynchronous BANs, automata that find themselves synchronously unstable are automata that influence each other. The interaction digraph G^* of $\mathcal{N}^*$ accounts for none of the corresponding

23256 influences. This shows that synchronism in (B)ANs only maps onto the notion of every day life simultaneity if very strong implicit assumptions are made on the meaning of the rest of the formalisation. In an intrinsically discrete model such as BANs, ruling it out altogether cannot be done without squarely dismantling the model.

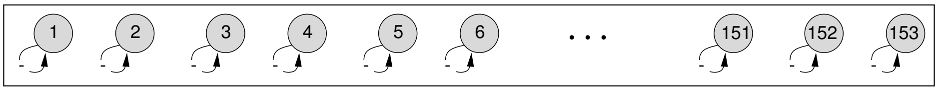


Fig. 2 – Interaction graph $G^* = (V^*, A^*)$ of BAN $\mathcal{N}^* = \{f_i : x \mapsto \neg x_i, \forall i \in V^*\}$, where $V^* = \{i \leq 153\}$.

Modelling a real system with BANs implies (1) possible synchronism in local automata state changes, (2) synchronism in automata instabilities, and (3) unavoidable synchronism making our observer's attention coincide specifically with the network configurations that we observe, take interest in, and model – which are supposed, under the dynamical systems view on BANs, to mark the network's state at the exact instants when both, changes of interest to us *begin* to occur, and the potential for these changes (instabilities) suddenly (dis)appears – rather than with configurations in the interim that we might be unaware of and certainly are not modelling, and which might not mark the beginning of anything special. The third source of synchronism gives their whole meaning to discrete modelling and the comment of about change on page 2. Since BANs offer several media for synchronism, they can be argued to take their meaning at a finer (lower) level of abstraction than the one at which "simultaneity in nature" makes sense. They cannot be soundly deprived of one sort of synchronism without being deprived of others, or completed with a consistent justification of the difference in treatment.

Time Flow

Systems we consider are often assumed to be conditional to properties of time (flow). Despite this, their formal definitions sometimes allows properties of time flow to mingle or overlap with their own properties. So time flow also determines what properties we attribute to a system. But it isn't always clearly distinguished from causality. And generally, when it isn't regarded as a pre-existing constraint on the systems we consider, time flow – or just "time" – is seen as a "resource", suggesting that we have a tacit obligation to use it sparingly, and, without fail, in finite quantity. Whatever we call it, we tend to assume that time flow pre-exists both the systems we study and the attention we invest in them, and that it frames the systems' behavioural possibilities and the leeway we take on them. Operational Research works its satisfaction and optimisation problems around it ; Bio-informatics builds models out of what it knows of it or despite what it doesn't [84–88] ; and even for Concurrency, time flow is mostly something *in* which distributed pieces of computation can be reunited [82, 83]. Despite its very dense presence in the scientific landscape, time flow is seldom is a *primary* object of our researches. The lack of deliberate scientific interest leaves plenty space for spontaneous interpretations, and for a natural tendency to expediently distinguish, confuse, overlook or classify issues and properties related to time flow such as simultaneity, synchronicity, precedence, subsequence, difference, determinism, periodicity, causality, time scale, change, process of change, realism, duration. Thus we usually miss out on (a) the vicariance of some of these properties for which time flow might actually not be the exclusive medium, and (b) possible leeway through this vicariance. Again, consider *the possibility of synchronism in a formal network* ($|U(x)| > 1$). The absence of known causality it conveys (*cf.* page 5) is a typical example of lack of information traditionally getting outshined by specialised knowledge and by the assumptions drawn out of it. If anything, what that lack of information represents is "wriggle room". Results [81] suggest that under very specific conditions, attributes of time flow might participate in the overall network computation in ways that are comparable to logical gates. Synchronism brings together otherwise presently independent pieces of information and creates a new relation between. This new line of research is one that my project proposes to pursue.

Generally, throughout my research, I propose to consider systematically (a) abstraction-induced lack of information (*cf.* page 3) and (b) possible, (implicitly) assumed properties of time flow. The BAN formalism makes it possible to compare effects of (a) and (b) on the one hand, with effects of properties specific to networks on the other. Thus, I can work towards a better understanding of network sensitivity to (a), and clarify of the kind of information time flow is a vehicle of – eventually the kind of computation complexity it can manage (*cf.* O6 on page 12).

1.1 Project-related publications

References in bold font and framed by superscripted asterisks (*e.g.* *[35]*) denote accepted or published papers. Other references (*e.g.* [89]) denote unpublished papers.

[35] **Combinatorics of Boolean automata circuits dynamics**. J. Demongeot, M. Noual and S. Sené. Discrete Applied Mathematics, 160 :398–415, 2010.

[90] **Dynamics of circuits and intersecting circuits**. M. Noual. Proceedings of LATA'2012.

[89] **A combinatorial problem concerning binary necklaces and attractors of Boolean automata networks**. M. Noual. https://arxiv.org/abs/1605.01505 2013.

Although cycles in the interaction digraphs of ANs have been known since the 1980's to be the "engines" of the complexity of AN behaviours, their rigorous influence on AN behaviours and the way they interact with one another is is not so well known. Papers [35], [90] and [89] focus on two types of BANs, namely Boolean Automata Cycles (BACs) and Boolean Automata Double-cycles (BADs). A BAC (resp. a BAD) is a BAN whose interaction digraph is an isolated cycle (resp. two tangentially intersecting cycles). Using Number Theory, full characterisations of the dynamics of BACs [35] and of the asymptotic dynamics of BADs [90] under π are derived, and the combinatorial problem thereby solved is explicitly related [35] to other known combinatorial problems including some involving shift register machines [91] and Lyndon words [92]. A new substantially simplified formalisation of BAN dynamics under π in terms of group actions is introduced [90], together with the new notion of *order* $\omega(\mathcal{N})$ of a BAN which is much more informative and practical than the size n of $\mathcal{N}$ is to characterise its dynamics. For BACs and BADs, $\omega(\mathcal{N}) = \mathcal{O}(n)$ is easily expressed as a function of n [90]. Using Word Theory [92], the exlicit formulae counting the number of attractors of BACs [35] and of BADs [90] are compared to the corresponding formulae for positive BACs of same order [89]. This way, (i) tight upper bounds on the number of attractors $T(\mathcal{N})$ of BACs and BADs are provided, and (ii) elementary operations are evidenced that turn an arbitrary BAC/BAD $\mathcal{N}$ into a positive BAC $\mathcal{N}^+$ of same order, straightforwardly simulating $\mathcal{N}$. Ratio $\xi(\mathcal{N}) = T(\mathcal{N})/\omega(\mathcal{N})$ is introduced to convey the degree of freedom of $\mathcal{N}$, *i.e.* its propensity to behave in both numerous and in various ways [89]. Comparing BADs with BADs, it is proven that making cycles that are otherwise isolated intersect causes an exponential reduction (w.r.t. n and $\omega(\mathcal{N})$) of the BANs' number of attractors without reducing (much) the order $\omega(\mathcal{N})$, *i.e.* it causes a decrease of $\xi(\mathcal{N})$. As shown in [89], this means that it brings the mean attractor period closer to the maximal possible value of an attractor period, implying that it is mostly the small, and conjecturally the most unstable attractors that are filtered out by the intersection. The ensuing insights of the three papers are the following. (1) *When nothing else than the network's clockworks are causing the occurrence of local changes* (*i.e.* under π), *autonomous cycles of interactions allow for a great degree of global freedom. The cycles act like space on which a great range of different information can alternatively be stored.* (2) *Networks tend to lose degrees of freedom as their underlying structural cycles become more intricately intersected.* (3) *The largest, possibly most stable and thereby less synchronism-allowing attractors of BACs and BADs are the most numerous.*

[93] **General transition graphs and Boolean circuits** M. Noual. http://hal.archives-ouvertes.fr/hal-00452025/fr/ 2010.

In [93], I introduced General Transition Graphs (GTGs) : the digraph $(\mathbb{B}^n, T)$ representing *all* possible transitions of $\mathcal{N} = \{f_i\} : (x, y) \in T \Leftrightarrow \forall i,\ y_i \in \{x_i, f_i(x)\}$. Possibly because of their size, but mostly for reasons given at the top of page 1, GTGs hadn't yet been studied at all in the context of ANs. Paper [93] fully characterises the GTGs of BACs using State Transition- and Word Theory-like techniques and some Number Theory. It shows that $|U(x)|$ serves as a potential function for BACs. The insight this paper provides is the following. *On autonomous cycles, a punctual amount of asynchronism* (actually, *of precedence*) *suffices to stabilise for good local instabilities that would otherwise last.*

[94] **Asynchronous Dynamics of Boolean Automata Double-Cycles** T. Melliti, M. Noual, D. Regnault, S. Sené, J. Sobieraj. Proceedings of UCNC 2015, LNCS 9252 :250–262.

This paper fully characterises the transient and asymptotic dynamics of asynchronous BADs. To do so, it introduces new tools to formalise BAN trajectories as algorithm executions. This gives an efficient way of considering updates in BANs and a nice understanding of how information is relayed.

[36] **Disjunctive networks and update schedules**. E. Goles, M. Noual. Advances in Applied Mathematics, 48 :646–662, 2012.

Block-sequential update schedules (BSUSs) are deterministic and periodic update schedules that F. Robert [32] introduced in the 80's and that have been the object of much attention since then [20, 25, 50, 95] [36, 38, 49]. In [49], the dynamics of BACs under all BSUSs were compared to the fully characterised dynamics of BACs under π [35]. As in [35, 90], explicit formulae were given for all BACs under all BSUSs. Paper [36] follows Paper [49]. It studies more generally how the choice of update schedule impacts on the dynamics of an arbitrary BAN $\mathcal{N}$. It does so by expliciting the BAN $\mathcal{N}'$ whose dynamics under π is exactly the dynamics of $\mathcal{N}$ under the given BSUS. In addition, it proposes a classification of Disjunctive Boolean ANs (DANs) according to the robustness of their dynamics with respect to changes of their update schedules. For this classification, BSUSs are considered as well as a relaxed version of BSUSs inspired by [96], namely, *fair update schedules* (FUSs). Finally, comparing the effects of BSUSs and FUSs, Paper [36] proves that in a synchronism-allowing context, redundancy in the updating can impact significantly on a BAN's behaviour. This result differs from one proven in [97] about the lack of influence redundancy in the updates has when a sequential update schedule is assumed. Insights provided by Paper [36] are : (a) *The impact of updating redundancy is related to synchronism.* (1) *The possibility of manipulating the update schedule gives a tremendous lot of leeway on the BAN's dynamics.* (2) *Sequentialisation acts as a road-block, momentarily preventing information to access certain parts of the BAN.* (b) *Sequentialisation makes the processing of information more effective in terms of quantity of local changes, but possibly less effective in terms of successive steps.*

[81] **Synchronism *vs* asynchronism in Boolean networks**. M. Noual. http://arxiv.org/abs/1104.4039 2011. Research report gathering content later published in the proceedings of AUTOMATA'2012 [52], and content conditionally accepted for publication in Natural Computing [98].

This paper gives an exhaustive list of all the kinds of effects that the addition of synchronism to an otherwise asynchronous monotone BAN can have on its behavioural possibilities. Non-monotone BANs are not included in the study because, precisely, asynchronous non-monotone BANs can be simulated by asynchronous monotone BANs, and non-monotony is in itself the object of study of a central part of my research. Paper [81] proves which (structural) conditions a BAN must necessarily satisfy to be "significantly" sensitive to synchronism, and thereby proves that most of the time, the addition of synchronism does not result in a significant change. However, Paper [81] shows that sometimes it does. And when it does, a property sharing aspects with the non-monotony of f_is is involved. And contrary to a very widespread misconception about synchronism, synchronism then *stabilises* the network in a way asynchronism cannot.

[99] **Shortest Trajectories and Reversibility in Boolean Automata Networks**. M. Noual. http://arxiv.org/abs/1606.02613 2016.

This paper takes interest in the notion of *reversibility*. It calls *long trajectory* any trajectory $\mathcal{T} = (x(t))_{t\leq \ell}$ of a BAN $\mathcal{N}$ which has length ℓ greater than the size n of $\mathcal{N}$. A long trajectory $\mathcal{T}$ from $x = x(0) \in \mathbb{B}^n$ to $y = x(\ell) \in \mathbb{B}^n$ necessarily updates some automata more than once. So some of the local changes $x_i \rightsquigarrow f_i(x) = \neg x_i$ that $\mathcal{T}$ makes must be *reversed* by $\mathcal{T}$ ($\neg x_i \rightsquigarrow x_i$). And in the end, in y, part of the information about the history of the local changes made by $\mathcal{T}$ to get from x to y is lost. Paper [99] considers *shortest* trajectories of BANs between arbitrary configurations x and $y \in \mathbb{B}^n$. It introduces useful notions of *cause* and *potential*. With these notions, it determines in most cases and conjectures in the remaining cases what is the maximal length ℓ of the shortest trajectories of an arbitrary BAN. In doing so, it defines the "non-monotone effect" as a BAN property which is needed to have long shortest trajectories, *i.e.* that is necessary to have the capacity of

substantially exploiting reversibility to make a global change that cannot be made without making and reversing some local changes. And it conjectures a range of different ways this property can implement itself. The most basic of these ways is *as a property of the local update functions* $f_i : \mathbb{B}^n \to \mathbb{B}$. Another way, given in page 3 and Fig.1 (b), replaces the non-monotony of f_i's by some observational and time flow related conditions. Paper [99] also gives support to the following idea. *Except in non-monotone BANs, there tends to be a strong limit on the number of change reversions that are needed to get from one configuration to another*. The results of Paper [99] suggest the introduction of a notion of recursivity to characterise sensitivity to reversibility.

[100] **Towards a theory of modelling with Boolean automata networks - I. Theorisation and observations**. M. Noual, S. Sené. http://arxiv.org/abs/1111.2077 2011.

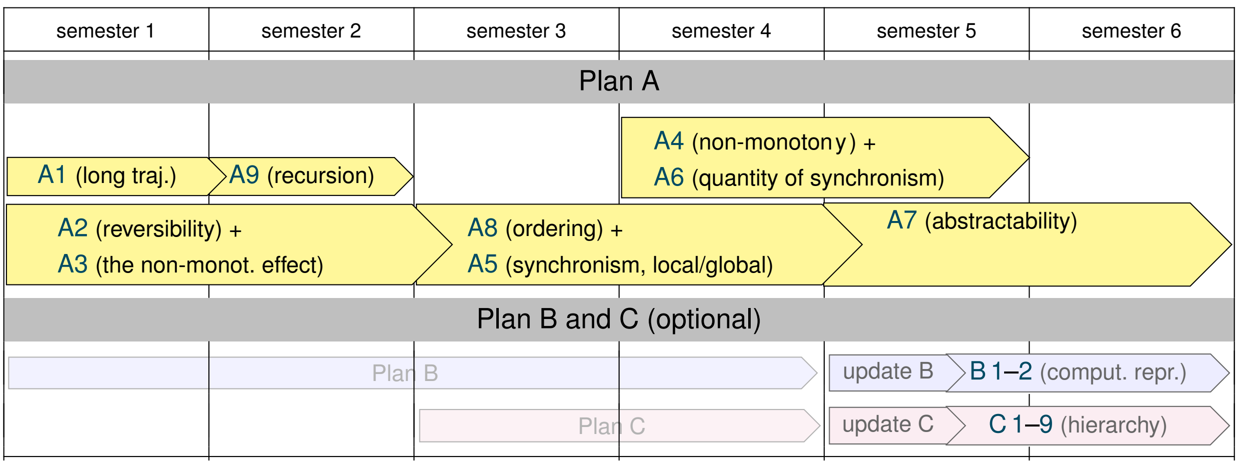
This paper takes a careful look at the BAN formalism. It raises and partially answers the questions *Which features of a model effectively carry immediate modelling meaning ? Which result from the initial task of formalisation of observations of reality into a mathematical language ?*

2 Objectives and Work Program

2.1 Anticipated total duration of the project : 3 years.

2.3 Work programme including proposed research methods

My research program is divided into three "plans" detailed in Section 2.5. Plans B and C are backup plans. I mention them because I expect them to serve as very appropriate alternatives if, or whenever Plan A calls for a temporary rest and some relevant change of perspective. Compared to Plan A, the scopes of Plans B and C are more narrow and the notions they focus on are now more settled. For these two reasons, Plans B and C are quite straightforward to implement and draw results from. However, their main purpose is precisely to emphasise in a practical way a type of fundamental understanding that Plan A means to develop and refine much further than it actually is. Plan A is the plan I wish to pursue in priority. It is more novel and has a wider scope. The results it aims at are also supposed to be immediately impactful and applicable. I expect them to quickly suggest significant updates or alternatives to Plans B and C. The table below represents an evolutive time schedule proposal putting emphasis accordingly on Plan A. In the arrows, labels A1, A2. . . B 1. . . C 1. . . refer to specific series of problems described in Section 2.5. The idea is to aim at the presentation of my work in a conference or publication at the end of each yellow arrow.



Tools

All my research uses elements of graph theory (because of its attention to the interaction digraph G) as well as Boolean Function Theory [101]. In addition to that, Plan A mainly uses State Transition Systems and Rewriting Theory. Locally, it might also use some inspiration from Computer Arithmetic to understand what causes shortest trajectories of BANs to be long (*cf.* A9). Plan B is inspired by Shift Register Machines [91]. It relies on notions related to this formalism to represent network computation in meaningful and familiar ways. Boolean Function Theory is essential to it. Just like the work presented in [35, 89, 90] in the direct lines of which Plan C is set, Plan C substantially relies on Combinatorics on Words [92] and Number Theory. It might also borrow tools from Information Theory (*cf.* C2) and Symbolic Dynamics [102] which is a very closely neighbouring field waiting to be explicitly related to ANs more than it already discreetly has in [89].

Approach

I will illustrate the approach of Plan A with an example. In a BAN's interaction structure G, *negative cycles* (aka *negative feedback loops*) are oriented cycles that have an odd number of negative arcs (*cf.* Fig.1). They are known in some (asynchronous) settings to be responsible for the asymptotic oscillations of BANs [27, 55, 103–105]. In other settings (allowing for synchronism), negative cycles are not necessary to have asymptotic oscillations [106, 107] [35, 90]. Asymptotic oscillations can therefore not intuitively be attributed to negative cycles, nor to synchronism. A closer look at oscillating BANs reveals that with or without negative cycles, the generating process of asymptotic oscillations works essentially the same way. "Something" disallows the collapse of an offset between the actual state of a certain automaton i and the pending influence sent out by this same automaton i to itself, possibly via the intermediary of other automata j. This "something" can be (a) a negative cycle, or (b) the combination of (b1) a positive cycle, (b2) some *in situ* potentiality already contained in the network's initial configuration $x = x(0) \in \mathbb{B}^n$, and (b3) some synchronism.

Generally, depending on the specific context, an arbitrary effect observed in a BAN – such as asymptotic oscillations – may have different implicants (conditions or properties implying it). There necessarily is a property or "generating mechanism" that is common to all implicants. At worst/best, the common property is the effect itself. In this (unlikely) case, the implicants' respective implications of the effect provide what Plan A targets : an atomic explanation and understanding of the effect. Plan A proposes a systematic approach consisting in the following steps. (S1) Select a notable global effect E exhibited by networks (oscillations, multiple fixed points, long or recursive trajectories, reversibility, trajectorial vicariance, sensitivity to precedence. . .). (S2) Identify the range of ways W that effect E can be implied in different settings under different conditions. (S3) Explore the relationship between those ways W in order to (S4) draw a common ability/generating mechanism/cause C. Then in turn, (S5) identify the different possible implementations I of C, and (S6) identify the range of effects E' that are implied by implementations I. In general, it is not always very clear at first glance whether an arbitrary network property or feature (*e.g.* to have asymptotic oscillations, to have a negative cycle, non-monotony, "the non-monotone effect") should be seen rather as an effect or as a cause. This might depend on the context. The property might be given slightly different meanings depending on it. The property might also be a facade for a composite of more subtle properties that aren't put to use exactly the same way in different contexts. Plan A means to address all these concerns. And thus, considering a certain network property P that is commonly put forward in the literature, it must first (S0) determine at which step of the cyclic procedure can P be brought in for study. For instance, the BAN property of non-monotony is one drawing attention to itself. So [81] naturally raised the problem of identifying what the effect of non-monotony *per se* actually is. Two closely related effects are (E1) synchronism-sensitivity [81], and (E2) substantial use of reversibility inducing long shortest trajectories [99]. Both effects are easy to produce with a non-monotone BAN. Both effects can also to some extent be reproduced with a monotone BAN. But to do so still requires a form of non-monotony [81, 99]. Plan A means to study further the causes and effects of non-monotony (*cf.* A3), *not* to explore the concept of non-monotony for itself, but to explore the more subtle and unknown mechanisms that it accounts for.

This approach is significantly inspired by Recursion Theory. It means to better understand and define the computational capacity of network-ness. It is based on the construction of an exhaustive collection of elementary (although refinable) abilities of networks which can be combined to *allow for and explain most notable global behavioural possibilities of networks* – *i.e.* abilities/causes proven to be expressive enough to reformulate the necessary conditions implying the effects (*cf.* step (S2)). Present candidates for this collection are the following. (C1) The ability to make a choice. (C2) Memory or the ability to recycle information and sustain a rhythm. (C3) The ability to "notice" a difference between two things such as the experience of a change of state and the communication of it – "notice" meaning *to have the said difference per se result in something* (rather than having something result in something different, or having different things impact differently). (C4) Recursion. (C5) Stabilisation and destruction of potential for change. Among several possible implementations of those abilities, perhaps the most unsurprising are respectively : (I1) A positive cycle, (I2) & (I3) A negative loop on an automaton i acting as both the seat and the witness of the difference $x_i = x_i(t) \neq f_i(x) = x_i(t+\tau)$, (I4) Non-monotonous interactions caught in a cycle, (I5) Precedence of effective updates (*i.e.* updates of unstable automata) in agreement with the direction of arcs in the interaction structure of the network. By means of a large set of relatively small and local questions and problems, Plan A offers to proceed with the examination of these candidacies and the search for finer ones. Understanding elementary abilities, their strict effects and possible implementations is certainly a definite step towards identifying the building blocks of an arbitrary network computation. At this very early stage of the development of a theory of network-ness, the objective can however not yet be to settle for a fixed (partial) basis like primitive recursion is for computational theory. An exploratory approach like Plan A is first needed.

2.2 Objectives

Uncovering common generating mechanisms serves a number of purposes :

- O1 It produces understanding of networkness which is not only more rigorous, but also easier to manipulate intuitively. Since it relies less on specifics it also is more portable. Since it concentrates on effects that are essentially scalable, it uncovers causes that also are scalable. Incidentally, the intrinsic simplicity of the BAN formalism already justifies in itself the use of this formalism because it offers an exclusive possibility of properly isolating fundamental mechanisms and effects. But since BANs simulate ANs, the scalability of properties studied justifies it even further.
- O2 Uncovering common generating mechanisms better prepares us to deal with new cases in which we observe the same effects without any of the implicants we are used to.
- O3 Through O1, my research effectively materialises a possibility of organising the literature's plethora of results concerning ANs. In the form of a progressive collection of common meaningful notions structured by a backbone of essentially non-specific mechanisms, it provides middle grounds for these results to be formally related, unified and pooled.
- O4 My research answers questions about interaction systems that can only arise in a context where neither a specific model nor a specific interpretation needs to be taken seriously.

It is a progressive approach to understanding networkness, based on an essentially unstable notion of causality. With this particular approach, the formal expression (in terms of implications) of causal relationships as we currently understand them consistently comes accompanied by indications on how this underlying understanding of ours could eventually become obsolete and in demand of a more precise formulation. And it concretely suggests directions in which our understanding can be pushed to finer, more expressive levels of abstraction. Indeed, the answers this approach provides consistently raise new questions, starting with : *What other ways are there to implement the same effect ?* – a question that is good protection against deadlocked approaches and perspectives resulting from our starting point. And when this approach reveals the involvement of one property (*e.g.* synchronism) in the generation of a given effect (*e.g.* a stabilisation of local instabilities that asynchronism cannot settle, *cf.* summary of [81] on Page 8), it raises the question of how this relates to the (alleged) involvement of the same property in other mechanisms (*e.g.* synchronism's alleged responsibility in entertaining local instabilities under BSUSs). Generally, this approach forces us to take unprecedentedly close look at notions that are the objects of traditionally settled questions and cannot possibly draw any attention to themselves in a context that already has fixed its interest onto a fully defined interaction system, real or formal. The apparent contradiction between (1) the fact that synchronism added to asynchronous monotone BANs only really impacts by stabilising local instabilities [81], and (2) synchronism's apparent and notorious role in entertaining local instabilities under BSUSs is typically the kind of hitch that I am looking for. And in the light of [81], this particular hitch actually pointed out that synchronism is not a fine enough notion to be used to explain either of these effects. On the one hand, following the comments of pages 5 and 6, this hitch suggests that there might be no such thing as *sensitivity to synchronism*, nor *to asynchronism* for that matter. *Sensitivity to precedence of causally related events* seems to be a much more meaningful way of coining the same effect. On the other hand, the hitch suggests that synchronism might simply *not* have the capacity to entertain and/or stabilise instabilities. And for that matter, asynchronism might not either. The hitch emphasises all the other differences between BSUSs and the asynchronous updating constraint that might be held responsible for entertaining local instabilities instead of, or in addition to synchronism. Periodicity, or perhaps even more subtly the specific kind of redundancy inherent to it, is one of them. In the lead of F. Robert [32], a great many studies have been supporting the general idea that "update schedules have great influence on the dynamics of BANs" [20, 96, 106] [15, 36, 38, 49]. Notwithstanding this, until periodicity's involvement *per se* in this influence is deliberately studied, there is no rigorous way to form an intuitive understanding of what causes the entertainment of local instabilities under BSUSs and what, other than asynchronism, tends to prevent the entertainment of local instabilities when synchronism is not exploited.

O5 This project's approach to networkness aims at answering fundamental questions, *e.g.* (i) *What influence do cycles have on behavioural properties of ANs ?*, an early question raised more than 30 decades ago [103] that has never stopped feeding publications since [27, 55, 104, 105, 108], and (ii) *What influence does non-monotony have on behavioural properties of ANs ?*, a more recent question brought up in [81] by time-related concerns.

O6 This project's approach instigates the laying of foundations for what can be called a "Fundamental Theory of Time Flow" (*cf.* Section *Time Flow*).

O7 When a given cause has several possible implementations (*cf.* (a) and (b) on page 9), then, depending on the context, one of them might have reasons to be considered to be more elaborate than another (*e.g.* (a) might be considered to be more elaborate than (b) because it involves negative interactions). Being aware of other possible implementations producing the exact same effects naturally helps keep in mind the possibility that when the more elaborate implementation is encountered, there might be more subtle atomic mechanisms operating at a lower level of abstraction than the one we are at. Moreover, this project's approach produces insight on the kind and range of mechanisms that those might be.

O8 Further than O3 and in lines with it, this project's approach builds up a solid basis for an ambitious and longer term objective which is to lay the foundations of a transdisciplinary axiomatisation of modelling (of systems) consistent both with the mathematical theory of modelling and with the *apparent* multitude of scientific conceptions of modelling. An initiating step of this initiative has already been proposed in [15, 100]. The formal basis that the present project can provide for it makes it even more realistic. Of course, once this basis provided, the development of the initiative needs to happen in parallel of some invested transdisciplinary dialogue As of today, it is however not too early to outline its two main motors. (1) A certain shift of attention from specifics and realism to definition and consistency. (2) The systematic endeavour to refine and update our scientific views so that instead of speaking of *theory and formalism* as opposed to *reality and nature*, we rather speak of objects that are *abstractions* of one another with a sense of the term "abstraction" that is effectively explicitable on a case-by-case basis.

O9 More immediately, the approach of this project suggests the design of canonical representations of network structures to evidence the core computation networks are making in agreement with the fundamental understanding we have of that (*cf.* Plan B). And using elementary *scalable* building blocks (*cf.* O1) for these representations (i) opens a possibility of defining a *formal* notion of *approximation* of one representation of a network by a another (*cf.* Plan C), and in the same way, (ii) suggests a formal definition of *abstraction level* and an attendant notion of *emergence complexity* accounting for well-defined effects enabled between well-identified levels of abstraction (*cf.* Plan C), as opposed to the emergence complexity often used to account for mostly opaque and unidentified informal causality.

O10 My research approach emphasises fundamental concepts – what we understand of the causal relationships between them, as well as what we don't. In that, it can inspire and guideline the choice of new relevant mathematical restrictions of the formal systems that we commonly study, so as to avoid the kind of restrictions that are both too strong and too intuitively intractable to be plausibly viable in the long run (*cf.* page 4).

O11 In the same way, the reseach I propose can be critically practical to get reliable grips on mathematical assumptions/restrictions that are already made. More than just encouraging a fine understanding of (a) how these assumptions relate to other independently existing properties of the formalism, (b) how the assumptions and their relations all combine to allow the derivation of the targeted results, and (c) what in the assumptions precisely *isn't* put to use, this approach prioritises this understanding. And thus, it directly favours insights on possible ways of generalising existing results drawn under specific assumptions and restrictions.

O12 Instead of studying models under specific defining assumptions and restrictions, my approach consists in studying the assumptions, restrictions and definitions themselves. It can therefore also take care of decisive generic steps of the modelling process : (M1) Check the *consistency* of the assumptions of a model with its formalisation, its defining hypotheses, its intended interpretation, and its other assumptions (*cf.* Section *Illustrative discussion. . .*). Separate clearly interpretations from formalisations and possibly identify confusions and misconceptions. When appropriate, considering the biological understanding of a system and of *how* the system is actually modelled (especially at what level of abstraction), provide counter-examples to evidence incompatibility between parts of this understanding that are juxtaposed or that are abstractions of one another [81]. (M2) Evaluate the consequence or inconsequence of an assumption (*cf.* summary of [81] on Page 8). (M3) Based on (M1) and (M2), decide whether or not, and *how* to take into account informal assumptions such as "simultaneity in nature is fortuitous". (M4) Based on (M1) and (M2), evaluate the reliability of the modelling allowed for by an established model. (M5) Settle the balance between : (i) the requirement of rigour that guarantees control over the subsequent interpretations of results derived from the model and with great likelihood limits the model's representational capacity, and (ii) the requirement of prompt efficiency of the modelling that aims at a more ambitious representation of complexity consistent with the modeller's idea of realism. (M6) Check for differences in observations and in formal derivations that might be attributable to differences of levels of abstraction (*cf.* A7). (M7) Explore lack of information and lack of known causality, where there is an increased risk of ambiguity and misinterpretation and also possibly an opportunity of exploitable flexibility. (M8) Based on (M6) and (M7), suggest possible changes of perspective on the real system. Notably, these tasks promote an essentially informatician understanding of (biological) systems and information, as opposed to a mathematical understanding of the system's models, backed up or not with bio-mathematical understanding of some rudiments of the biological modelling. Of computer scientists, they require to drop the role of "flunky illustrator of experimental data collected upstream" in order to assume responsibility for parts of reasonings that have been considered the exclusivity of the "applied" sciences, possibly even call into question the "applied" scientists' understanding on their own grounds of their own knowledge.

2.4 Data handling : not applicable

2.5 Other information : Work Plans A, B, and C

Plan A

Plan A consists of a progressive and abundant collection of well-bounded problems of which a non-exhaustive and summarising sample figures below. Because these problems are so tightly connected to one another, for the most part, I will address them in parallel. And here, I will motivate them with a single starting question : When we know that configuration $y \in \mathbb{B}^n$ is reachable from configuration $x \in \mathbb{B}^n$, when can we also know through which (asynchronous) trajectory y can effectively be reached from x (if through any) ? A direct trajectory is one along which no automaton changes states more than once. For most monotone BANs (MANs), if there is a synchronous transition from x to y, *i.e.* if $HD(x, y) = \{i \leq n \,|\, x_i \neq y_i\} \subset U(x)$, then there is a direct asynchronous trajectory from x to y. So in this case, the difference between x and y informs on all the changes that must have been made to get from x to y [81].

A 1 LONG SHORTEST ASYNCHRONOUS TRAJECTORIES. ■ Prove or disprove the conjectures of [99] : depending on the type of BAN (non-monotone, nice, totally positive. . .), determine the maximal length ℓ of a long shortest trajectory from x to y, and from x to an attractor. The proofs of the results called for by Problem A1 are expected to further specify the effect of non-monotony and non-monotony's relation to reversibility (*cf.* summary of [99] on Page 8), and thus significantly help if not resolve the next problem.

A 2 REVERSIBILITY. ■ Identify conditions under which reversibility is needed to perform a global change $x \in \mathbb{B}^n \rightsquigarrow y \in \mathbb{B}^n$. ■ Identify couples $(x, y) \in \mathbb{B}^n \times \mathbb{B}^n$ such that there exists only indirect (reversibility-using) trajectories from x to y. ■ Determine to what extent this is a property attributable to x and y rather than to the BAN. ■ Study sensitivity to reversibility by adding possibilities of using it to BANs that otherwise are forbidden to. Characterise the trajectories that are affected by this addition, their starting points and their ending points. Determine BANs for which addition of reversibility only impacts by elongating trajectories.

A 3 THE NON-MONOTONE EFFECT. ■ (Dis)prove the implication of the non-monotone effect in "need for reversibility" (*cf.* summary of [99] on Page 8). ■ (Dis)prove that in its absence, only two atemporal factors determine the final outcome of a trajectory $\mathcal{T}$: (1) *in situ* information contained in the initial configuration x, and (2) an atemporal choice possibly implemented through precedence, made somewhere along $\mathcal{T}$, determining which of the initial in situ information is to be used and survive, and which will eventually disappear without leaving any trace on y. ■ Explicit the remaining ways of implementing the non-monotone effect.

A 4 NON-MONOTONY, MORE GENERALLY. Let us consider totally positive acyclic BANs (PANs) as references. The "computational capacity" (*cf.* page 10) of PANs is minimal in the sense that their only ability is to transfer information. The idea of Problem A4 is to compare non-monotone BANs (NANs) to PANs and determine what mechanisms need to be added to PANs to make them reproduce the notable behavioural possibilities that we observe in NANs and tend to attribute the responsibility of to non-monotony (intrinsic effects of non-monotony that cannot be reproduced without non-monotone f_i's, *i.e.* effects that represent the real difference between NANs and MANs, are strongly related to time flow and can be expressed in terms of the efficiency of the execution of mechanisms). Let us call "effects of non-monotony" these possibilities and "mechanisms of non-monotony" what needs to be added to PANs to reproduce these effects without non-monotone f_i's. Some notable, proven or conjectured mechanisms of non-monotony are the following : (a) a *systematic* interaction between the inputs of a given automaton prior to this automaton's use of its inputs, (b) contradictory parallel paths dealing differently with the same piece of information, or simply, (c) contradictory initial *in situ* potential, (d) non-planarity of G or anything that can cause information pathways to cross, (e) negative cycles or anything that can produce the effect of restocking information by recycling old pieces of information into new, possibly contradictory ones, (f) consistent dissociation between a change and the order it sends out or translates into. Mechanisms (a), (d) and (e) especially relate to (respectively) synchronism sensitivity [81], reversibility, and recursion (*cf.* A9). ■ Compare all these mechanisms, their alternative implementations and the expressivity they enable. ■ Identify the sort of non-monotone effect(s) that synchronism sensitivity is attributable to [81] and deduce sufficient conditions for a BAN to be sensitive to synchronism, or narrow down the necessary conditions given in [81].

A 5 LOCAL AND PUNCTUAL *VS* GLOBAL AND GENERAL. ■ Study the information that a punctual possibility of synchronism (a.k.a. a degree of punctual local freedom) conveys about the BAN's future, especially about the number and diversity of its attractors (a.k.a. a degree of global freedom/ambiguity).

Determine how sequentialisation turns punctual local freedom into global freedom. Determine whether a punctual possibility of synchronism always translates into (i) having several ways of producing the same global change which conveys a form of trajectorial robustness, or whether it may translate into (ii) global ambiguity. ■ Identify sequential event orderings with the special power to entertain synchronism.

A 6 QUANTITY OF SYNCHRONISM. Non-monotony can be considered as an abstraction of more elementary subtle non-monotone mechanisms. Any NAN can be simulated by a MAN under specific timing conditions on updates. ■ (Dis)prove that sensitivity to the quantity/degree of synchronism, *i.e.* sensitivity to $|U(x)|$ requires a form of non-monotony, *i.e.* an elaborate or composite mechanism.

A 7 "ABSTRACTABILITY". Under a BSUS, a transition $x \in \mathbb{B}^n \to y \in \mathbb{B}^n$ represents a series of changes that starts in x and eventually leads to y after having updated each automaton exactly once, not necessarily one at a time, not necessarily all at once either. On the contrary, in the asynchronous transition graph, and similarly in the GTG (*cf.* Page 7), transitions $x \to y$ only represent changes that are already possible in x. Generally, as emphasised in [15], the choices of updating constraints we make determine the degree of "atomicity" of the transitions we consider. This points towards a very significant difference between various traditional AN frameworks. ■ Study the effect of the non-atomicity of transitions on the BAN's apparent behaviour. Compare the (general) transition graphs of BANs with their "contracted versions" obtained by alleviating atomicity. Determine the degree of non-atomicity allowed before "substantial information" on how the BAN works is lost, or before enough information to recognise non-ambiguously a specific mechanism (*e.g.* a negative cycle, a non-monotonous f_i) is lost. ■ Study and compare uniform contractions of certain intensity with or without the restriction to asynchronous transitions. ■ Relate sensitivity to non-atomicity with sensitivity to ordering. ■ Determine how "abstractability", *i.e.* the possibility to skip steps of a trajectory without loosing essential information about the BAN's clockworks, relates to : (i) the degree of "vertical-non-monotony", *i.e.* the degree of sequentialisation needed to reveal non-monotony (*cf.* Fig.1 (b)), ■ (ii) sensitivity to ordering, and ■ (iii) the maximal degree of possible synchronism $|U(x)|$. In particular, (iii) is to determine the extent to which abstractability requires the lack of causality conveyed by possibility of synchronism.

A 8 ORDERING. ■ Study sensitivity to ordering of events/updates and distinguish that from sensitivity to precedence *per se* if possible.

A 9 RECURSION. Exploring ways of implementing typical Computer Architecture mechanisms (*e.g.* counters made of half-adders) in the main classes of BANs (non-monotone, nice, totally positive. . . [99]) has recently lead me to define a new notion of "fundamentally recursive trajectory" (FRT). ■ With this definition and the notion of potential defined in [99], show that for a FRT to have recursion depth D, it requires at least 2^D different potential (versions) to have visited the same automaton. ■ Characterise the main classes of BANs with the depth of the recursion that they allow, or explicit what more than recursion is needed to do so. ■ Compare the non-recursive trajectories of NANs with the trajectories of MANs. ■ Determine to what extent non-recursivity is a property of the trajectory rather than one of the BAN.

Plan B

Plan B works towards producing constructive and modular representations of networks.

B 1 ■ Define a canonical way of representing an arbitrary BAN $\mathcal{N}$ by an equivalent BAN $\mathcal{N}^*$ (equivalent in the sense that the transition graphs of $\mathcal{N}$ and $\mathcal{N}^*$ are isomorphic) such that (i) automata that merely relay information, automata that merely negate and relay it, automata that merely copy and distribute information, and automata that actually compute, all appear clearly distinct from one another in the interaction graph G^* of $\mathcal{N}^*$, and (ii) the complexity of the computation in terms of the nestedness and arity of the functions f_i involved in $\mathcal{N}$ is also evidenced structurally in $\mathcal{N}^*$.

B 2 ■ On those grounds, define a notion of computational complexity for BANs in structural terms and ■ compare BANs from the first levels of complexity, *e.g.* compare the canonical representations of BACs, BADs and straightforward generalisations of those.

Plan C

C 1 ■ Complete [35, 89, 90] with a study of non-monotone BADs under π, *i.e.* explicit the total number of attractors, the number of attractors of each period and the order of any non-monotone BAD.

C 2 ■ With an appropriately defined notion of entropy, consider the following (possibly incorrect or incomplete) insight in the case of BACs and BADs : *adding cycle intersections reduces the entropy of the BAN*, and prove it or formalise the reason why there is no appropriate notion of entropy allowing to prove it.

C 3 ■ Formalise a notion of *structural simplicity* $\delta(\mathcal{N})$ to compare BANs of same order $\omega(\mathcal{N})$ and allow the proof of : *for any BAC or BAD* $\mathcal{N}$ *and positive BAC* $\mathcal{N}^+$ *of same order,* $\delta(\mathcal{N}) < \delta(\mathcal{N}^+)$ *holds*. Prove it.

C 4 Formalise general(isable) notions of : ■ *simulation* relating BAN behavioural possibilities, ■ *similarity*, in terms of comparable BAN sizes and orders for instance, ■ *representation* of one BAN $\mathcal{N}$ by a structurally simpler BAN $\mathcal{N}'$, imposing similarity between $\mathcal{N}$ and $\mathcal{N}'$, and further relating their automata, mechanisms and structures, ■ *approximation* of a BAN $\mathcal{N}$ by a BAN $\mathcal{N}'$ that represents and simulates $\mathcal{N}$. Choose the definitions of these notions so that the following can be proven : (i) $\mathcal{N}'$ *represents* $\mathcal{N}$ *implies that* $\mathcal{N}'$ *simulates* $\mathcal{N}$ and (ii) *for every BAC and BAD* $\mathcal{N}$*, there is a positive BAC* $\mathcal{N}^+$ *of same order that approximates* $\mathcal{N}$*.* ■ Prove (i) and (ii) or explicit formal reasons why it cannot be done.

C 5 ■ Simultaneously define some *elementary operations* (*e.g.* pinching a cycle to turn it into a double-cycle) or *contractions* (*e.g.* the Graph Theoretic concept of minor) in order to prove that (iii) *The BAN* $\mathcal{N}'$ *obtained by application of these elementary operations is an approximation of* $\mathcal{N}$. Prove (iii) or explicit formal reasons why it cannot be done. If (iii) can be proven, then the approximation relation defines a hierarchical classification $\mathcal{H}$ of BANs of same order : BANs nearer to the top of $\mathcal{H}$, able to simulate a greater number of other BANs of same order, are structurally simpler, and have more degrees of freedom/ambiguity (greater $\xi(\mathcal{N})$, *cf.* Page 7).

C 6 ■ Study the effect of more intricate intersections than tangential intersections, and using the definition of representation, find "good" representatives of BANs in which they appear (*e.g.* BANs with strongly connected and planar interaction structures) to determine what behavioural possibilities they allow.

C 7 ■ Study behavioural properties of BANs with regards to $\mathcal{H}$. Introduce a definition of *emergence* to distinguish properties that are *not* scalable w.r.t. $\delta(\mathcal{N})$ and w.r.t. the levels of $\mathcal{H}$ – *e.g.* the property conveyed by $\xi(\mathcal{N})$ of a certain diversity of asymptotic behaviour produced with a fixed order (*cf.* summary of [89]).

C 8 ■ Study what becomes of recurrent configurations and attractors when tracing up $\mathcal{H}$.

C 9 ■ To complement and/or support Plan B, based on $\mathcal{H}$, define a *grammar* of elementary, scalable BANs that can be combined through elementary scalable BAN connections to produce a range of (all ?) notable BAN behaviours (possibly biologically pertinent [41] or computationally complete).

2.6 Descriptions of proposed investigations involving experiments on humans, human materials or animals : not applicable

2.7 Information on scientific and financial involvement of international cooperation partners : not applicable

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4 Requested modules/funds

Reason:	Cost in euros:
Module Temporary Positions for Principal Investigators: Postdoctoral researcher Dr Mathilde Noual, full-time for 36 months TV-L E13	$66\,600 \times 3 =$ 199800
Travel expenses: - One international trip and stay per year (including one to Chile's Universidad Adolfo Ibáñez, and at least two to the SFBT's annual seminar) - Short visits to collaborators in Europe and conferences attendance - Contribution to invitations of collaborators	$2500 \times 1 + 1000 \times 2$ $+800 \times 3$ $+800 \times 3 =$ 9300
Publication expenses: conference registration fees	$500 \times 3 =$ 1500
Total	210 600

5 Project Requirements

5.1 Employment status information

Noual, Mathilde, 18-month term postdoctoral fellow at the Freie Universität Berlin

5.2 First-time proposal data

Noual, Mathilde

5.3 Composition of the project group: not applicable

5.4 Cooperation with other researchers

5.4.1 Researchers with whom I have agreed to cooperate on this project

Prof H. Siebert, Prof A. Bockmayr, Dr H. Klarner.

5.4.2 Researchers with whom I have collaborated scientifically within the past 3 years

J. Aracena, L. Calzone, J.-P. Comet, J. Demongeot, M. Kaufman, H. Klarner, T. Melliti, A. Naldi, D. Regnault, A. Richard, S. Sené, E.H. Snoussi, J. Sobieraj, D. Thieffry.

5.5 Scientific equipment: not applicable

5.6 Project-relevant cooperation with commercial enterprises: not applicable

5.7 Project-relevant participation in commercial enterprises: not applicable

6 Additional information: not applicable